

ETH-TH/2000-05  
March 1999

# Electroweak Physics, Theoretical Aspects

Z. Kunszt

*Institute for Theoretical Physics, ETHZ  
CH-8093 Zurich, Switzerland*

## Abstract

I discuss two aspects of the electroweak interactions: the status of the precision measurement of the electroweak parameters and their impact on the Higgs search at future colliders.

# 1 Precision calculations

During the last decade we witnessed an impressive progress at LEP, SLC and Tevatron achieved by collecting an enormous amount of electroweak data on the Z and W bosons and their interactions [1, 2]. This allows for an unprecedented precision test of the Standard Model at the level of the per mil accuracy. At this precision one and two-loop quantum fluctuations give measurable contributions and an interesting upper limit on the mass of the Higgs-boson can be obtained.

## 1.1 Input values

In the Standard Model at tree level the gauge bosons  $\gamma, W, Z$  and their interactions are described in terms of three parameters: the two gauge coupling constants  $g, g'$  and the vacuum expectation value of the Higgs-field  $v$ . We need to know their values as precisely as possible. They have to be fitted to the three best measured physical quantities of smallest experimental error:  $G_\mu, M_Z$  and  $\alpha$ . The muon coupling  $G_\mu$  is extracted from the precise measurement of the muon life-time using the theoretical expression

$$\frac{1}{\tau_\mu} = \frac{G_\mu^2 m_\mu^5}{192\pi^3} \left(1 - \frac{8m_e^2}{m_\mu^2}\right) \left[1 + 1.810 \left(\frac{\alpha}{\pi}\right) + (6.701 \pm 0.002) \left(\frac{\alpha}{\pi}\right)^2 + \dots\right] \quad (1)$$

It is crucial that the electromagnetic corrections to the muon life time are finite and that they are known up to next-to-leading order accuracy. The  $\mathcal{O}(\frac{\alpha_s}{\pi})$  term has been obtained by van Ritbergen and Stuart only very recently [3], it lets reduce the theoretical error with factor of two. Equation 1 gives a unique correspondence between the muon life-time and  $G_\mu$  since the non-photon corrections are all lumped into its definition. As a result  $G_\mu$  can be considered as a physical quantity. Using the measured value [4] we get

$$G_\mu = (1.16637 \pm 0.00001) \times 10^{-5} \text{ GeV}^{-2}. \quad (2)$$

The value of  $M_Z$  is extracted from the line shape measurement at the Z-pole. There are subtleties in the theoretical definition of the mass and the width at higher order associated with the truncation of the perturbative series and gauge invariance. The latest best value is [5]

$$M_Z = (91.1871 \pm 0.0021) \text{ GeV}. \quad (3)$$

Finally, the best value of  $\alpha$  is extracted from the precise measurement of the electron anomalous magnetic moment  $(g_e - 2)$  [4]

$$1/\alpha = 137.03599959 \pm 0.00000038. \quad (4)$$

I recall the leading order relations

$$G_\mu = \frac{1}{\sqrt{2}v}, \quad M_Z = \frac{1}{2 \cos \theta_W} gv, \quad \alpha = \frac{g^2}{4\pi} \sin^2 \theta_W, \quad \tan \theta_W = \frac{g'}{g}. \quad (5)$$

Additional physical quantities like the mass of the W-boson  $M_W$ , the lepton asymmetries at the Z-pole, the leptonic width of the Z-boson  $\Gamma_l$  etc. are derived quantities. At the level of the per mil accuracy the predictions obtained in Born approximations for derived quantities, however, fail significantly.

## 1.2 Quantum corrections

The precision test of the Standard Model is obtained by confronting the measured values of derived quantities with the precise prediction of the theory. Since the Standard Model is a renormalizable quantum field theory [6] the theoretical predictions of the theory can be improved systematically by calculating higher order corrections. In particular, the recent precision of the data requires the study of the complete next-to-leading order corrections, resummation of large logarithmic contributions and a number of two loop corrections. At higher order the derived quantities show sensitivity also to the values of the mass parameters  $m_t, M_H, m_b$  and the QCD coupling constant  $\alpha_s$ . From direct measurements one obtains  $\alpha_s = 0.119 \pm 0.002$ ,  $m_t = 173.8 \pm 5.0 \text{ GeV}$  and  $m_b = 4.7 \pm 0.2 \text{ GeV}$ ,  $M_H \geq 102 \text{ GeV}$ . The error bars give parametric uncertainties in the predictions and limit our ability to extract a precise value of the Higgs mass. The calculation of the higher orders requires a choice of the renormalization scheme. The on-shell scheme can be regarded as the extension of the well-known scheme of renormalization in QED, it uses as input  $\alpha, M_Z, M_W, M_H$  and  $m_f$ . In the  $\overline{\text{MS}}$ -scheme the measured values of  $\alpha, G_\mu, M_Z, m_f, \alpha_s$  are used to fix the input parameters of the theory with  $M_H$  as free parameter. The  $\overline{\text{MS}}$  gauge couplings evaluated at the scale of  $M_Z$  are denoted as  $\hat{e}$  and  $\hat{s}^2 = \sin^2 \hat{\theta}_W(M_Z)$ . The on-shell definition of the mixing angle  $s^2 = \sin^2 \theta$  is given by the tree level relation  $s^2 = 1 - M_W^2/M_Z^2$  and, therefore, it is a physical quantity. The renormalized parameters  $\hat{s}^2, \hat{e}^2$  can be completely calculated in terms of  $G_\mu, \alpha$  and  $M_Z$ . It is customary to define auxiliary dimensionless parameters.  $r_W$  is defined by the relation

$$s^2 c^2 \equiv \frac{\pi \alpha}{\sqrt{2} G_\mu M_Z^2 (1 - \Delta r_W)}. \quad (6)$$

It is a physical quantity and gives the radiative corrections to the  $M_W$ . The asymmetries measured at the Z-pole are given in term of the effective mixing angle

$$\sin^2 \theta_W^{eff} = \frac{1}{4} \left( 1 - \frac{\bar{g}_{Vl}}{\bar{g}_{Al}} \right) = s^2 (1 + \Delta k'), \quad A_{\text{FB}}^l \equiv \frac{3 \bar{g}_{Vl}^2 \bar{g}_{Al}^2}{(\bar{g}_{Vl}^2 + \bar{g}_{Al}^2)^2} \quad (7)$$

where the dimensionless parameter  $\delta k'$  is again defined in terms of physical quantities. The leptonic width depends on the vector axial vector coupling and on the corrections to the Z-propagator. This requires the introduction of the so called  $\rho$ -parameter

$$\Gamma_l = \frac{G_\mu M_Z^3}{6\pi\sqrt{2}} (\bar{g}_{Vl}^2 + \bar{g}_{Al}^2) \rho \quad (8)$$

These type of auxiliary functions can be calculated in different renormalization schemes. The corrections  $\Delta r_W, \Delta k', \Delta \rho$  (and a number of additional useful dimensionless quantities) are known in various schemes and play an important role in the analysis of electroweak physics, because they give the precise predictions of the theory for simple observables as  $M_W$ , the leptonic asymmetries *etc.* in terms of  $\alpha, G_\mu$  and  $M_Z$ . It is very useful to have the results in different schemes since it allows for cross-checking the correctness of the result and to estimate the remaining theoretical errors given by the missing higher order contributions. The electroweak radiative corrections are dominated by two leading contributions: the running of the electromagnetic coupling and large  $m_t$  effects to  $\rho$  ( $\Delta \rho_t \approx 3 G_\mu m_t^2 / (8\pi^2 \sqrt{2})$ ).

### 1.3 Running electromagnetic coupling

Because of gauge invariance the running of  $\alpha$  is completely given by the photon self-energy contributions

$$\alpha(M_Z) = \frac{\alpha}{1 - \Delta\alpha} \quad (9)$$

where

$$\Delta\alpha = -\text{Re}(\hat{\Pi}^\gamma(M_Z^2)) = -\text{Re}(\Pi^\gamma(M_Z^2)) + \text{Re}(\Pi^\gamma(0)) . \quad (10)$$

The self-energy contribution is large ( $\approx 6\%$ ). It can be split into leptonic and hadronic contributions

$$\Delta\alpha = \Delta\alpha_{\text{lept}} + \Delta\alpha_{\text{had}} \quad (11)$$

The leptonic part is known up to three loop

$$\Delta\alpha_{\text{lept}} = 314.97687(16) \times 10^{-4} \quad (12)$$

and the remaining theoretical error is completely negligible. The hadronic contribution is more problematic since it can not be calculated theoretically with the required precision since the light quark loop contributions have non-perturbative QCD effects. One can extract it, however, from the data using the relation

$$\begin{aligned} \Delta\alpha_{\text{had}} &= \frac{\alpha}{3\pi} M_Z^2 \text{Re} \int_{4m_\pi^2}^{\infty} \frac{R_{e^+e^-}(s')}{s'(s' - M_Z^2 - i\epsilon)} \\ R_{e^+e^-}(s) &= \frac{\sigma(e^+e^- \rightarrow \gamma^* \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} \end{aligned} \quad (13)$$

Conservatively, one calculates the high energy  $\sqrt{s} \geq 40 \text{ GeV}$  contribution using perturbative QCD and the low energy contribution  $\sqrt{s} \leq 40 \text{ GeV}$  is estimated using data [7]. Unfortunately, the precision of the low energy data is not good enough and the error from this source dominates the error of the theoretical predictions

$$\Delta^{(5)}\alpha_{\text{had}} = 0.02804 \pm 0.00064, \alpha^{-1}(M_Z) = 128.89 \pm 0.09 . \quad (14)$$

One can, however, achieve a factor of three reduction of the estimated error assuming that the theory can be used down to  $\sqrt{s} = m_\tau$  when quark mass effects can be included up to three loops. Such an analysis is quite well motivated by the successful results on the tau lifetime. In the hadronic vacuum polarization the non-perturbative power corrections appear to be suppressed and the unknown higher order perturbative contributions are relatively small. In this theory driven approach the error is reduced to an acceptable 0.25% value

$$\alpha^{-1}(M_Z) = 128.905 \pm 0.036 . \quad (15)$$

It is unlikely that the low energy hadronic total cross section will be measured in the foreseeable future with a precision leading to essential improvement.

## 1.4 Comment on the muon anomalous magnetic moment

I note that the hadronic vacuum polarization contribution to the muon anomalous magnetic moment  $a_\mu \equiv (g_\mu - 2)/2$  is more problematic as a result of a different weight factor in the dispersion integral

$$a_\mu^{\text{had}} = \left( \frac{\alpha m_\mu}{3\pi} \right) \left[ \int_{4m_\pi^2}^{E_{\text{cut}}^2} \frac{R_{e^+e^-}(s)^{\text{data}}(s)K(s)}{s^2} + \int_{E_{\text{cut}}^2}^{\infty} \frac{R_{e^+e^-}(s)^{\text{pQCD}}(s)K(s)}{s^2} \right] \quad (16)$$

where we splitted the perturbative and low energy contributions.  $K(s)$  is a kinematical weight factor which together with  $1/s^2$  enhances the low energy contributions. As a result the experimental error of the measured value of the hadronic vacuum polarization contribution leads more than 1% error in the theoretical prediction. It is expected that high statistics data collected in DAΦNE in the future will reduce this error with a factor of two. Such an improvement is very well motivated in view of the experimental effort of the ongoing Brookhaven experiment which will achieve a precision of  $\approx 40 \times 10^{-11}$ , a significant reduction in comparison with the present error of  $\approx 730 \times 10^{-11}$  (see [9] and references therein). Note, however, that the hadronic contribution from light-to-light scattering diagrams cannot be measured and the theoretical estimates have large uncertainties leading to a theoretical error of  $\approx 40 \times 10^{-11}$ . Accepting this estimate with the precise measurement of  $a_\mu$  it will be possible to test for the present of anomalous coupling (SUSY) contributions provided they are large ( $\approx 100 \times 10^{-11}$  or larger). It is unlikely that one gets improvements over the existing LEP limits.

## 1.5 Higher order corrections to $M_W$ and the mixing angle

As we noted above, the simplest physical observables for precise test of the Standard Model are  $M_W$  and the  $\sin^2 \theta_W^{\text{eff}}$ . It is convenient to consider the radiative corrections in the  $\overline{\text{MS}}$  scheme where with good accuracy  $\sin^2 \theta_W^{\text{eff}} \approx \hat{s}^2$ . It is given in terms of the input parameters via the relation

$$\hat{s}^2 \hat{c}^2 = \frac{\pi \alpha(M_Z)}{\sqrt{2} G_\mu M_Z (1 - \hat{r}_w)} \quad (17)$$

where  $\hat{r}_w = 0$  in leading order. Using the measured value of  $\sin^2 \theta_W^{\text{eff}}$ ,  $M_Z$  and  $G_\mu$  we obtain a value  $\hat{r}_W = 0.0058 \pm 0.000480$  different from zero at the  $12\sigma$  level. If one carries out a similar analysis for  $M_W$  the evidence for the presence of subleading corrections is even better. The radiative correction  $\hat{r}_W$  does not contain the large effect from the running  $\alpha$  but it receives large custodial symmetry violating corrections because of the large top-bottom mass splitting

$$\Delta \hat{r}_W|_{\text{top}} = -c^2/s^2 \Delta \rho \approx 0.0096 \pm 0.00095 \quad (18)$$

Subtracting this value we get about  $6\sigma$  difference coming from the the loops involving the bosonic sector (W,Z,H) and subleading fermionic contributions. At this level of accuracy many other corrections start to become important and the the size of errors coming from

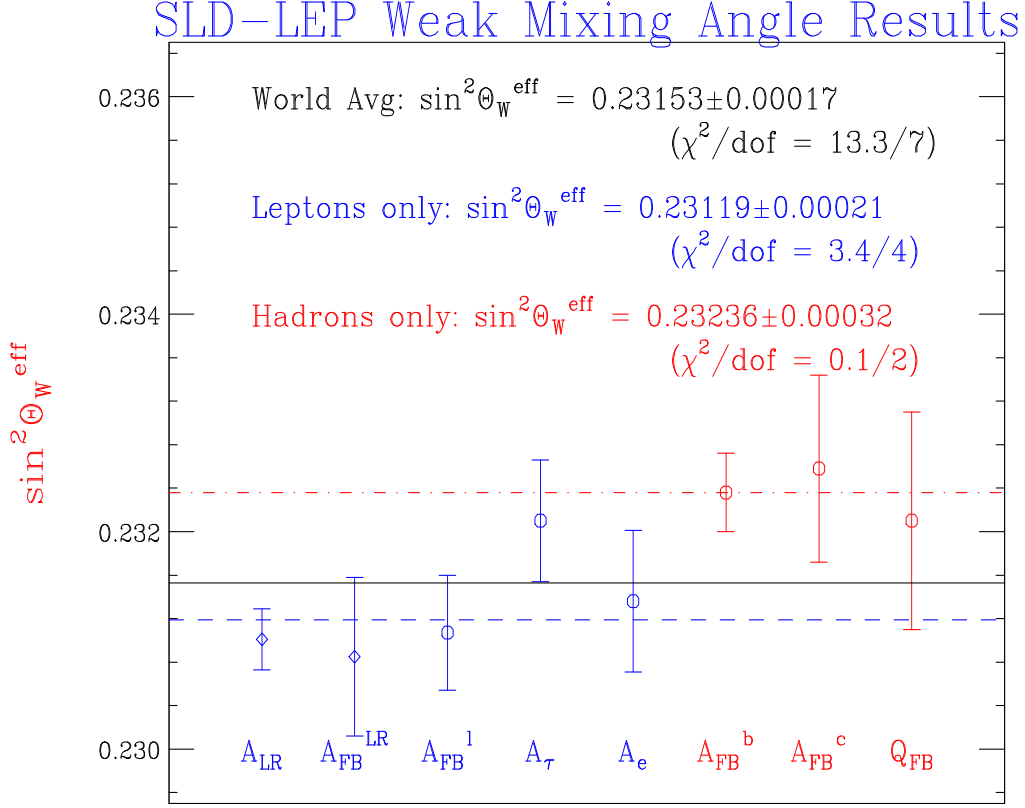


Figure 1: Summary of all the determinations of  $\sin^2 \theta_W^{\text{eff}}$  [1].

the errors in the input parameters leads to effects of the same order. In particular, we get some sensitivity to the value of the Higgs-mass. Beyond the complete one loop corrections it was possible to evaluate all important two loop corrections:  $\mathcal{O}(\alpha^2 \ln(M_Z/m_f))$  corrections with light fermions, mixed electroweak QCD corrections of  $\mathcal{O}(\alpha\alpha_s)$ , two loop electroweak corrections enhanced by top mass effects of  $\mathcal{O}(\alpha^2(m_t^2/M_W^2)^2)$  together with the subleading parts of  $\mathcal{O}(\alpha\alpha_s^2 m_t^2/M_W^2)$  and the very difficult subleading correction of  $\mathcal{O}(\alpha^2 m_t^2/M_W^2)$ . It is remarkable that last contribution proved to be important in several respect [8]. Its inclusion reduced significantly the scheme dependence of the results and lead to a significant reduction of the upper limit on the Higgs mass.

## 1.6 Global fits

This summer the LEP experiments and SLD could finalize their results on the electroweak precision data. The most important development is that the final value of SLD on the leptonic polarization asymmetry was reported which implies  $\sin^2 \theta_W^{\text{eff}} = 0.23119 \pm 0.00020$ . A nice summary of the results is given in Figure 1 [1]. According to a recent analysis of the EWW working group [5] the new world average is

$$\sin^2 \theta_W^{\text{eff}} = 0.23151 \pm 0.00017 \quad \text{with} \quad \chi^2/\text{d.o.f} = 13.3/7. \quad (19)$$

This gives only rather low confidence level of 6.4%. The origin of this unsatisfactory result is the  $2.9\sigma$  discrepancy between the values  $\sin^2 \theta_W^{\text{eff}}$  derived from the SLAC leptonic polarization

asymmetry data and from the forward backward asymmetry in the b-b channel at LEP and SLC. The results obtained from a global fit to all data give somewhat better result but there we are hampered with the problem that the polarization asymmetry parameters disagree with each other with  $2.7\sigma$ , therefore, the  $\chi^2$  is relatively large.

## 1.7 The weak charge of the atomic Cesium

Recently, a new determination of the weak charge of the atomic Cesium (via studying the  $6s \rightarrow 7s$  parity violating tensor transition) has been presented [10]

$$Q_W(^{133}_{55}\text{Cs}) = -72.06 \pm (0.28)_{exp} \pm (0.34)_{th} \quad (20)$$

with considerable improvement with respect to earlier results

$$Q_W(^{133}_{55}\text{Cs}) = -71.04 \pm (1.58)_{exp} \pm (0.88)_{th}. \quad (21)$$

In the theory,  $Q_W$  measures the product of the vector and axial vector neutral current coupling of the u and the d quarks  $C_{1u}, C_{1d}$

$$Q_W = -2 [C_{1u}(2Z + N) + C_{1d}(Z + 2n)] \quad (22)$$

A crucial feature of this test is that it constrains the value of the parameter  $\epsilon_3$  [12]

$$Q_W = -72.87 \pm 0.13 - 102\epsilon_3^{\text{SM}} \quad (23)$$

with  $\epsilon_3^{\text{SM}} = 0.0053 \pm 0.0013$  for  $M_H = 70 - 1000$  GeV. According to the data [11]

$$Q_W^{exp} - Q_W^{th} = 1.28 \pm 0.46 \quad (24)$$

a three standard deviation effect. One should accept this result with some care in view of the significant reduction of the experimental error. It would be important to cross-check this result with other independent experiments. Also the estimate of the theoretical uncertainties coming from atomic physics calculation may be too optimistic. In ref. [11] the deviation was attributed to the existence of a non-sequential  $Z'$ -boson. and the data have been used to constraint its properties.

## 2 Constraints on the Higgs mass

### 2.1 Upper limit from the measured value of $M_W$

The final results of the electroweak radiation corrections for  $M_W$  and  $\sin^2 \theta_W^{eff}$  can be parameterized in terms of the input parameters including their errors in simple approximate analytic form [8]. For example in the  $\overline{MS}$ -scheme one obtains for the W-mass

$$\begin{aligned} M_W = & 80.3827 - 0.0579 \ln\left(\frac{M_H}{100}\right) - 0.008 \ln^2\left(\frac{M_H}{100}\right) \\ & - 0.517 \left( \frac{\delta\alpha_h^{(5)}}{0.0280} - 1 \right) + 0.543 \left[ \left( \frac{m_t^2}{175} - 1 \right) \right] \\ & - 0.085 \left( \frac{\alpha_s(M_Z)}{0.118} - 1 \right) \end{aligned} \quad (25)$$

where  $m_t$ ,  $M_H$  and  $M_W$  are in GeV units. This formula accurately reproduce the result obtained with numerical evaluation of all corrections in the range  $75 \text{ GeV} \leq M_H \leq 350 \text{ GeV}$  with maximum deviation of less than 1 MeV. Using the world average of the measured values of the W-boson mass  $M_W = 80.394 \pm 0.042 \text{ GeV}$  [13] (with input parameters  $\alpha_s = 0.119 \pm 0.003$ ,  $m_t = 174.3 \pm 5.1 \text{ GeV}$ ,  $\delta\alpha^{(5)} = 0.02804 \pm 0.00065$  one obtains at 95% confidence level an allowed range for the Higgs mass of  $73 \text{ GeV} \leq M_H \leq 294 \text{ GeV}$ . Similar results exists also for  $\sin^2 \theta_W^{eff}$  extracted from the asymmetry measurements at the Z-pole with somewhat better (95% confidence ) limits of  $95 \text{ GeV} \leq M_H \leq 260 \text{ GeV}$ . Without global fits we got a semi-analytic insight on the sensitivity of the precision tests to the Higgs mass. We also see that the precise measurements of  $M_W$  have already provided us with competitive values in comparison with the those obtained from the measurement of  $\sin^2 \theta_W^{eff}$ .

## 2.2 Results from global fits

It is interesting that the values of the Higgs mass obtained in a recent global fit [14] are in good agreement with the simple analysis based on the value of  $M_W$  or  $\sin^2 \theta_W^{eff}$  as described above. From the global fit one obtains an expected value for the Higgs boson of  $160 - 170 \text{ GeV}$  with error of  $\pm 50 - 60 \text{ GeV}$ . The 95% confidence level upper limit is about  $260 - 290 \text{ GeV}$ .

## 2.3 Can the Higgs-boson be heavy?

The precision data can not rule out yet dynamical symmetry breaking with some heavy Higgs-like scalar and vector resonances. The minimal model to describe this alternative is obtained by assuming that the new particles are heavy (more than 0.5 TeV) and the linear  $\sigma$ -model Higgs-sector of the Standard Model is replaced by the non-renormalizable non-linear  $\sigma$ -model. It can be derived also as an effective chiral vector-boson Lagrangian with non-linear realization of the gauge-symmetry [15, 16]. How can we reconcile this more phenomenological approach with the precision data? Removing the Higgs boson from the Standard Model while keeping the gauge invariance is a relatively mild change. Although the model becomes non-renormalizable, but at the one-loop level the radiative effects grow only logarithmically with the cut-off at which new interactions should appear. In equation (25) the Higgs-mass is replaced by this cut-off The logarithmic terms are universal, therefore, their coefficients must remain the same. The constant terms, however, can be different from those of the Standard Model. The one loop corrections of the effective theory require the introduction of new free parameters which influence the value of the constant terms. The data, unfortunately, do not have sufficient precision to significantly constrain the constant term appearing in  $M_W$ ,  $\sin^2 \theta_W^{eff}$  and  $\Gamma_l$  (or alternatively in the parameters  $\epsilon_1, \epsilon_2, \epsilon_3$  [12] or  $S, T, U$  [19] ). In a recent analysis [17] it has been found that due to the screening of the symmetry breaking sector [18], alternative theories with dynamical symmetry breaking and heavy scalar and vector bosons still can be in agreement with the precision data up to a cut-off scale of 3TeV.



## References

- [1] M. L. Swartz, hep-ex/9912026 and references therein.
- [2] A. Sirlin, hep-ph/9912227 and references therein.
- [3] T. van Ritbergen and R. G. Stuart, hep-ph/9904240; Phys. Rev. Lett. **82** (1999) 488 [hep-ph/9808283].
- [4] C. Caso *et al.*, Eur. Phys. J. **C3**, 1 (1998).
- [5] LEP and SLD Electroweak Working Group, preprint CERN EP/2000-16,
- [6] G. 't Hooft, Nucl. Phys. **B35**, 167 (1971;); G. 't Hooft and M. Veltman, Nucl. Phys. **B44**, 189 (1972;)
- [7] F. Jegerlehner, hep-ph/9901386.
- [8] G. Degrossi and P. Gambino, hep-ph/9905472.
- [9] A. Czarnecki and W. J. Marciano, hep-ph/9810512.
- [10] C. S. Wood, S. C. Bennett, J. L. Roberts, D. Cho and C. E. Wieman, Can. J. Phys. **77** (1999) 7.
- [11] R. Casalbuoni, S. De Curtis, D. Dominici and R. Gatto, Phys. Lett. **B460** (1999) 135 [hep-ph/9905568].
- [12] G. Altarelli, R. Barbieri and F. Caravaglios, Nucl. Phys. **B405**, 3 (1993)
- [13] K. Mönig, these proceedings
- [14] G. D'Agostini and G. Degrossi, hep-ph/0001269
- [15] T. Appelquist and C. Bernard Phys. Rev. **D22** 200 (1980).
- [16] A. Longhitano, Phys. Rev. **D22**, 1166 (1980), Nucl. Phys. **B188**, 118 (1981).
- [17] J. A. Bagger, A. F. Falk and M. Swartz, hep-ph/9908327.
- [18] M. Veltman, Act. Phys. Pol. **B8**, 475 (1977).
- [19] M. Peskin and T. Takeuchi, Phys. Rev. Lett. **65**, 2963 (1990).